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The Magnetic Field and the Central Core of the Earth

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Summary

The suggestion that the inner core of the Earth is liquid and is the seat of a strong magnetic field is examined. Contrary to expectations of a magneto-hydrodynamic rigidity for the inner core, it is shown that a liquid inner core with a strong magnetic field should not transmit shear waves. The absence of the phase *PKJKP* in seismic records is favourable to the hypothesis of a strong magnetic field in a liquid inner core. If the jump in *P* wave velocity at the inner core boundary is due to a magnetic field, the strength of the field must be of the order of 5×10^6 G whether the field geometry is toroidal or random. A strong toroidal field leads to an apparent ellipticity of the inner core. No such ellipticity is observed, so that if the field in the inner core is toroidal, it must be less than 5×10^5 G. A random field does not lead to an ellipticity effect and no limit can be set on the magnitude of a random field.

The energy required for a magneto-hydrodynamic rigidity in the inner core is excessive. The magnetic energy is equally divided between the inner and outer cores. The hypothesis of a strong magnetic field in the inner core cannot explain the observed short period variations of the surface field.

1. Introduction

A seismic discontinuity at a depth of 5 121 km separates the inner core of the Earth from the outer core. The necessity for such a discontinuity in interpreting the velocity distribution within the Earth was first pointed out by Miss Lehmann. Miss Lehmann (1936) showed that the observations of apparent *PKP* phases in the shadow zone could be explained by an inner core in which the *P* wave velocity was significantly higher than the *P* velocity in the outer core. The careful analyses of Gutenberg & Richter (1938) and Jeffreys (1939 a, b) firmly established the existence of an inner core having a *P* wave velocity higher than that of the outer core. The velocity in the inner core varies little and has an average value of about 11·2 km/s. The velocity in the outer core immediately adjacent to the inner core is in doubt. Jeffreys (1952) indicates that at a depth of 4 982 km the *P* velocity suddenly starts to diminish with depth and decreases until the inner core is reached where there is a jump in velocity from 9·4 to 11·16 km/s. Gutenberg (1951) shows a discontinuous change in the slope of the velocity curve at 5 121 km

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depth with a rapid increase of velocity to the value of 11·15 km/s. Though the details are uncertain there is definitely an increase of P velocity of about 10 per cent at a depth of about 5 121 km over the value at the 4 982 km depth.

The inner core contains only about 0·75 per cent of the Earth's volume. The total mass of the inner core is very uncertain since the inner core contributes only a small proportion of the total mass and moment of inertia of the Earth. Bullen's (1953) two models place the density at 12·3 and 22·3 g/cm^3, thus adequately emphasizing the uncertainties. It is very probable that the mass of the inner core is not more than two to three per cent of the mass of the total Earth.

Though the inner core represents but a small fraction of the Earth, the nature of the inner core plays an important role in the interpretation of the Earth's magnetic field and of the large-scale motions of the Earth. Birch (1940) suggested that the inner core represents a crystalline solid phase of the material of the liquid outer core where the latter is mostly liquid iron. Later workers (Bullen 1950 a, 1953 a, b; Jacobs 1953, 1954; Uffen & Misener 1954) make similar postulates. The principal difficulty in accepting this interpretation has been the fact that no shear wave, the phase *PKJKP*, has been observed to traverse the inner core. Bullen (1950 b, 1953 a, b) constructed travel-time curves for the purpose of detecting *PKJKP*. Bullen (1956), Hutchinson (1955) and Burke-Gaffney (1953) have all searched for this phase without success.

Before proceeding with the consideration of an alternative hypothesis we summarize the evidence leading to the suggestion that the inner core is solid.

1. The velocity, V_K, of a P wave in the outer core is fixed by the ratio of the incompressibility k to the density ρ, $V_K = (k/\rho)^{\frac{1}{2}}$. If the inner core were liquid this would require that the incompressibility undergo a sharp jump at the inner core–outer core boundary. Bullen (1949, 1950) suggests that the velocity distribution within the Earth is consistent with the hypothesis that k and dk/dp vary smoothly even across the core–mantle boundary. A smooth variation of k across the outer core–inner core boundary is obtained if the inner core is solid with a P wave velocity $V_I = \{(k+\frac{4}{3}\mu)/\rho\}^{\frac{1}{2}}$, assuming a reasonable value for the rigidity μ. Birch (1952), using the observed P wave velocity and an extrapolated incompressibility for iron, finds a Poisson's ratio of 0·37 to 0·38 for the inner core. This value is perhaps high for most metals but the calculation neglects effects of composition, temperature and phase change.

2. The outer core is generally taken to be iron or iron–nickel. If the inner core were liquid, an element heavier than iron is required for a denser inner core. Generally accepted cosmic abundance tables show no abundant element heavier than iron or nickel.

3. The temperature gradient within the outer core is assumed to be adiabatic and therefore the change of temperature in the core is small. At the increased pressure existing at the inner core boundary the liquid phase of the outer core would no longer be stable relative to the solid phase assuming the melting point is raised by pressure.

Since the evidence on the inner core is principally qualitative, the nature of the inner core remains very much in doubt. The purpose of the present paper is to examine the inner core in the light of two recent suggestions. Cole (1957) presents the hypothesis that the inner core is liquid and is the seat of a magnetic field. The assumed field is sufficiently strong to impart a magnetohydrodynamic rigidity to the inner core. MacDonald & Knopoff (1958), on the basis of asymptotic relations for the Thomas–Fermi–Dirac equation of state and on abundance arguments, show that silicon is probably a major constituent of the Earth's core. An alloy of approximate composition $(Fe,Ni)_2Si$ meets the requirements imposed by both the equation of state and abundance considerations.

2. Seismic evidence on a magnetic field in the inner core

We first investigate the limits set by seismic observations on the magnetic field residing in the inner core. The theory of the interaction of hydrodynamic and magnetic phenomena in an electrical conducting fluid has been developed by Alfvén (1950). Knopoff (1955) extended the theory to the interaction between elastic wave motions and a magnetic field in an electrical conductor. We review that part of the theory applicable to the interaction of a seismic wave with a magnetic field.

We consider a plane seismic wave incident upon a plane discontinuity between two liquids of characteristic elastic wave velocities α_0 and α_1. The angle of incidence of the plane wave is ϕ and the angle of emergence is θ; the wave approaches the discontinuity from the side of the fluid having velocity α_0. The fluid having a velocity α_1 is assumed to be electrically conducting with a finite conductivity σ and density ρ and is permeated by a uniform magnetic induction **B** parallel to the discontinuity. No magnetic field is assumed on side 0.

The linearized equations of motion in the fluid case are

$$\left(\nabla^2 - \mu\sigma\frac{\partial}{\partial t}\right)\mathbf{F} = \sigma\left\{\nabla \wedge \nabla \wedge \left(\frac{\partial \mathbf{U}}{\partial t} \wedge \mathbf{B}\right)\right\} \wedge \mathbf{B}$$

where

$$\mathbf{F}/\rho = \alpha^2 \nabla\nabla \cdot \mathbf{U} - \partial^2 \mathbf{U}/\partial t^2, \text{ (Knopoff 1955)}$$

U is the displacement vector of a particle and μ is the magnetic permeability. We assume a harmonic motion

$$\mathbf{U} = U_0 \mathbf{e} \exp i\omega\{(x \sin\theta + z\cos\theta)/V - t\}$$

where the z direction is chosen normal to the interface and the x direction is taken in the direction of the magnetic field. **e** is a unit vector lying in the x–z plane. The complex phase velocity in the region containing the magnetic field is given by the solution to the equation

$$(V^2 - \alpha_1^2)(V^2 + iV_e^2) = V_h^2(V^2 - \alpha_1^2 \sin^2\theta).$$

V_e is the product of the frequency and the skin depth $V_e = (\omega/\mu\sigma)^{\frac{1}{2}}$. V_h is the velocity of the magnetohydrodynamic wave (Alfvén wave), $V_h = B/(\mu\rho)^{\frac{1}{2}}$. Solving for the phase velocities under the constraint of Snell's law and assuming a high electrical conductivity $V_e \ll \alpha_1$, we obtain two complex phase velocities

$$V_1 = \{\alpha_1^2 + V_h^2[1 - (\alpha_1/\alpha_0)^2 \sin^2\phi]\}^{\frac{1}{2}}$$

and

$$V_2 = \left\{\frac{i\alpha_1^2 V_e^2}{\alpha_1^2 + V_h^2[1 - (\alpha_1/\alpha_0)^2 \sin^2\phi]}\right\}^{\frac{1}{2}}.$$

The significant feature of this solution is that the second mode attenuates within its wavelength and would not be observed at any distance. Cole suggested that a strong magnetic field would lead to a shear wave because of the magnetic rigidity and that, even though the inner core were fluid, the phase J should be observed assuming the inner core were the seat of a strong magnetic field. This conclusion is incorrect. The phase J should never be observed even in the presence of a magnetic field. This feature of the solution is readily understandable on physical grounds. The magneto-hydrodynamic waves travel with velocity V_h, are propagated along the lines of force of the magnetic field, and are polarized transverse to the lines of force. The field is parallel to the plane discontinuity. By Snell's Law, the ray is bent toward the normal since V_h is ordinarily less than α_0. These two conditions on the ray are inconsistent and the severe constraint imposed on the signal leads to rapid attenuation.

The prediction that the phase *PKJKP* should never be observed if there is a magnetic field in a liquid inner core is consistent with observations. This feature is an advantage of an earth model with a liquid inner core over a model with a solid inner core.

The jump in P wave velocity at the 5 121 km discontinuity may give us some information concerning the magnitude of the magnetic field. We first assume that the inner core has the same composition as the outer core. At the interface, the elastic velocities are equal

$$\alpha_1 = \alpha_2 = \alpha$$

The velocity in the inner core is

$$V_I = (\alpha^2 + V_h^2)^{\frac{1}{2}}$$

where we neglect the problem of the variation of the magneto-hydrodynamic term with polarization and angle. At the bottom of the outer core the velocity is $V_{K'} = \alpha$. The observed jump in velocity can be attributed to the presence of a magnetic field perturbing the P wave velocity in the inner core. If we assume the velocity-depth curves are computed for normal incidence, then using $V_I = 11{\cdot}2$ km/s, and $V_K = 10{\cdot}5$ km/s, the induction B in the inner core is 5×10^6 G where the jump in velocity is due entirely to the presence of a magnetic field. The magneto-hydrodynamic velocity $V_h = 4$ km/s is a value consistent with that assumed by Cole, though Cole underestimated the needed magnetic induction by an order of magnitude. If the minimum velocity noted by Jeffreys in the region between 4 982 km and 5 121 km depth is associated with a field free region, the jump in velocity to the value at 5 121 km can only correspond to a field far in excess of 5×10^6 G.

The calculation of a magnetic induction depends on the assumption of identical composition across the discontinuity. Under this condition there is no obvious mechanism whereby a large magnetic field could be concentrated in the inner core and not be found in the outer core. One means by which a magnetic field could be stored in the inner core is to make the outer core a poorer electrical conductor than the inner core. The magnetic field will reside in the better conductor because of the smaller ohmic losses therein. A difference in chemical composition between the outer and inner core could produce the needed discontinuity in electrical conductivity. MacDonald & Knopoff (1958) show that a bulk composition of the core of $(Fe,Ni)_2Si$ is consistent with a chondritic meteorite model of the Earth and with the representative atomic number as fixed by seismic observations and the asymptotic relations for the Thomas–Fermi–Dirac equation of state. The chondritic meteorite model is in turn consistent with the heat flow measurements. In the presence of a gravitational field, a liquid of bulk composition $(Fe,Ni)_2Si$ might be expected to separate with iron and nickel concentrated inwardly relative to silicon; in turn nickel concentrates inwardly relative to iron. Such a separation could provide a conductivity difference of one order of magnitude between the inner and outer core with the silicon rich outer core having a lower conductivity than the nickel rich inner core.

If the compositions of the inner and outer core differ, we can no longer take $\alpha_0 = \alpha_1$. The variation of P wave velocity with atomic number is indicated by the asymptotic relations for the Thomas–Fermi–Dirac model (March 1955; Gilvarry 1957). We should expect that α_0 should be $2\frac{1}{2}$ to $3\frac{1}{2}$ per cent higher than α_1 if there is a difference of 3 to 4 atomic numbers for the materials in the outer and inner core. Since $\alpha_1 < \alpha_0$, under the assumption of higher silicon content in the outer core, the estimate of the magnetic induction of 5×10^6 G required to produce the P velocity jump is too low. Alternatively if a smaller field of less than 5×10^5 G is postulated for the inner core, then at pressures of 10^6 atm the velocity in liquid iron–nickel is greater than the velocity in iron silicide in opposition to the predictions of the asymptotic relations. We note that at one atmosphere the velocities in elements Fe, Cu, Zn and copper–zinc brasses have the order predicted by the asymptotic relations but Ni does not.

3. Apparent ellipticity of the inner core due to a magnetic field

A further test of the existence of a magnetic field in the inner core results if we suppose that the field has a toroidal geometry. Bullard & Gellman (1954), and Elsasser (1950, 1955) postulate that the field is mainly toroidal. A second model by Cole pictures the field lines as turbulent or randomly non-radial. The poloidal field is here ignored as being of only a secondary nature. The magnitude of the poloidal field at the surface of the inner core is about 60 G. Elsasser suggests a conversion efficiency between poloidal and toroidal fields of the order of 10^3 so that a toroidal field of the order of 6×10^4 gauss in the inner core is a possibility. Alternatively if the jump in seismic velocity at the inner core boundary is due to a magnetic field, then the strength of the field in the inner core is at least 5×10^6 G regardless of the toroidal or turbulent character of the field. It should be noted that if the field is turbulent, large Maxwell stresses in excess of 10^4 atm must be maintained against only the viscosity and Coriolis forces.

If the field is toroidal then the angular dependence of the phase velocity of *PKJKP* is important. In the case of a diametral ray through the equator of the toroid, the velocity is $(\alpha_1^2 + V_h^2)^{\frac{1}{2}}$. A ray on the axis of the toroid has a velocity α_1. The variation in velocity is equivalent to an ellipticity of a homogeneous inner core. The difference in velocity between an equatorial and axial ray is $V_h^2/2\alpha_1$ for $V_h \ll \alpha_1$. A uniform magnetic field of 5×10^5 G throughout the inner core would lead to a difference in travel times of axial and equatorial rays of 0·6 s, a time difference too small to be observed. A field of 5×10^6 G produces a maximum ellipticity correction of 50 s. A rough investigation of the times of *PKIKP* has shown no such ellipticity. We infer that seismic travel times place an upper limit to the magnitude of the magnetic field, provided the field is toroidal. No such limit is possible if the field is random.

A further bit of evidence on the inner core is Gutenberg's (1957) observation of a precursor to *PKIKP* with a time lead of 12 to 15 s. This precursor might be interpreted as a consequence of oblique incidence upon the inner–outer core boundary. We resolve the incident ray onto the plane containing the magnetic field and the normal to the discontinuity and onto the plane normal to the magnetic field. The first component traverses the inner core with velocity

$$\{\alpha_1^2 + V_h^2[1 - (\alpha_1/\alpha_0)^2 \sin^2\phi]\}^{\frac{1}{2}}$$

while the other component travels with a greater velocity $(\alpha_1^2 + V_h^2)^{\frac{1}{2}}$. The travel-time difference between the two components is 12 s at $\Delta = 140°$ provided the magnetic induction is of the order of 5×10^6 G. However, Gutenberg points out that the precursor is a high-frequency component with a period of about 1 s while the main *PKIKP* phase has a period of about 3 s. Since the velocity V_1 is not a function of frequency, the precursor must originate elsewhere than in the magnetic field, unless diffraction in the presence of a complicated geometry provides the observed dispersion.

4. Geophysical consequences

The hypothesis of a strong magnetic field in a liquid inner core has several geophysical consequences. First, since the inner core is now presumed liquid instead of solid, the temperature in the inner core is now above the melting point for iron at the pressure of the core. None of the difficulties obtained by Jacobs (1954) and Uffen & Misener in reconciling the assumed solidity for the inner core with the melting point information (Simon 1953) and the temperature gradients in the outer core (Valle 1952) obtains in the present case. The central temperature can now be specified to be in excess of 4 000° K.

The magnetic time constant $4\pi\mu\sigma a^2$ determines the lifetime of the magnetic field; a is the radius of the body. Reducing the radius of the magnetic region from that of the outer core to that of the inner core reduces the time constant given by Elsasser (1950) to about 5 000 years. If we state that circulatory motions are not expressly forbidden in the outer core, the hypothesized reduction in conductivity of the outer core reduces this magnetic time constant to about the same value. Thus if we started with an initial budget of mechanical energy in both cores the magnetic energies resulting from the circulation would be equally partitioned in the inner and outer cores. Thus some other mechanism, probably of a mechanical type, must act in addition to the electrical mechanism in order to store a strong magnetic field in the inner core.

The surface magnetic field undergoes short period variations with a period of 500 years (Chapman 1951). Let us assume that these variations originate in the inner core. If the time constant for the decay of a field in the outer core is 5 000 years, then the fluctuating poloidal field at the surface of the inner core must be e^{10} greater than that needed to produce the observed fluctuations in the absence of an outer core. In order for these short period variations to originate in the inner core, the non-poloidal field in the inner core must vary in magnitude by at least 10^7 G. There is no seismic evidence for fields in excess of 10^7 G. The jump in *P* wave velocity at the inner core boundary requires a magnetic field in the inner core of the order of 10^7 G. If the short period variations are to originate in the inner core a still stronger total field is required. The fact that the inner core shows no seismic ellipticity rules out a toroidal field of this strength. The energy required to maintain a random field of this magnitude is very large and there are no apparent sources for this energy. It thus seems unlikely that the inner core is liquid.

5. Summary

A model in which a liquid inner core is permeated by a strong magnetic field is not inconsistent with seismic observations or with abundance arguments. In such a model, the phase *PKJKP* has a vanishingly small amplitude and should not be observed. The phase *PKJKP* has not been found. Because of the small energy associated with this phase (Bullen 1951), it should be most difficult to detect, so that the absence of *PKJKP* cannot as yet be regarded as positive evidence for a liquid core. The discontinuity of the *P* wave velocity at the inner core, if interpreted solely as due to a magnetic field, requires a field of 5×10^6 G in the inner core whether the field geometry is toroidal or random. Observations of the lack of angular dependence of the velocity of the phase *PKIKP* rules out a field greater than 5×10^5 G if the field is toroidal. No limit can be set if the field is random. The *PKIKP* precursor of Gutenberg is not inconsistent with a field of 5×10^6 G in the toroidal model but should not be observed in the random model. The random model for the field requires Maxwell stresses high compared with probable viscous stresses and Coriolis forces. If the field in the inner core is toroidal, it must be less than 5×10^5 G. No limit can be set if the field is random but a figure of 5×10^6 G is indicated. These fields are too small to explain the observed fluctuations of the magnetic field; the inner core is very likely not liquid.

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